PLANAR TUMBLING OF BODIES WITH NONZERO AERODYNAMIC TRIM

DURING ATMOSPHERIC ENTRY

Final Technical Report
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by

Maurice L. Rasmussen

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SUMMARY

An approximate analysis is presented that investigates the effect of non-zero trim on the planar tumbling motion of a vehicle undergoing atmospheric entry. A simple extension of a previously analyzed zero - trim - angle model is used to investigate the motion. It is found when the initial tumbling rate and trim angle are both positive, or both negative, that critical values of these parameters exist for which tumbling motions that always are arrested are separated from those that are not. Quantitative results are obtained on the basis of the approximate analysis. The results of this analysis should serve as a basis for conducting a more precise numerical analysis.

INTRODUCTION

A number of investigators have considered the tumbling motions of aerodynamically stable bodies entering planetary atmospheres. Tobak (1) reviewed previous work and showed that, under certain simplifying assumptions for planar tumbling, the problem is reduced to an embryonic form that is described by the equation for Painlevé's fifth transcedent (2). This equation describes the tumbling motion, its arrest, and subsequent oscillatory motion. In addition, it provides an angle-of attack history that is sufficiently accurate for practical use.

Tobak obtained approximate analytical expressions for the oscillatory behavior of a vehicle governed by a sine-function pitching moment. Tobak and Peterson (3) obtained universal curves that describe tumbling arrest by means of numerical solutions of the Painlevé equation. Analytical results for tumbling arrest, valid near the minimum angle of arrest, were obtained by Rasmussen (4). This report concerns the extension of this work to include the effects of nonzero trim.

The asymmetrical aspects of nonzero trim introduce new and complicated features to the tumbling problem. Whereas a tumbling body with zero trim and a symmetric moment will always end up in a decaying oscillatory motion as it descends through the atmosphere, the tumbling body having nonzero trim may or may not end up in oscillatory motion, depending on the initial conditions. Two other types of motion are possible for bodies having nonzero trim. One of these is a tumbling rate that ultimately increases until structural restrictions are exceeded. In the other, the body ceases to tumble as in the symmetric moment case, but instead of beginning to oscillate, it commences to tumble oppositely at an increasing rate. The goal of the present s study is to delineate quantitatively, or at least qualitatively, the various regimes of motion and to discover approximate analytic results that portray the essential features of the problem.

NOMENCLATURE

A	reference area
a	$s/2\dot{\alpha}_i$, parameter defined by (6)
b	parameter in atmospheric density law
C_m	pitching - moment coefficient
C_{mo}	characteristic pitching - moment coefficient
F	function defined by (9)
f	pitching - moment function, defined by (1)
I	pitching moment of inertia
ℓ	reference length
q_i	initial dynamic pressure
s	bv_i, parameter defined by (4)
t	time
v_i	initial vertical component of flight velocity
x	$\kappa \exp(st/2)$, independent variable replacing time
x_p	value of x at tumbling arrest
α	pitch angle
α_i	initial pitch angle
$\dot{\alpha}_i$	initial pitching rate
α_p	value of α at tumbling arrest
α_t	trim angle
$(\alpha_t)_{cr}$	critical value of α_t
β	$\tan^{-1}(\dot{\alpha}_i/s)$
$(\beta)_{cr}$	critical value of β
γ	$\tan^{-1}(\dot{\alpha}_i/2s)$
κ	initial value of x, defined by (4)

FORMULATION OF THE PROBLEM

The approximations and analysis that go into deriving the Painlevé equation for planar tumbling are the same as given by Tobak (1) and will not be repeated here. The density of the planetary atmosphere is assumed to vary exponentially as $\rho = \rho_o \exp(-by)$. Here we assume that the pitching - moment coefficient has the form

$$C_m = C_{m_o} f(\alpha) \quad , \tag{1}$$

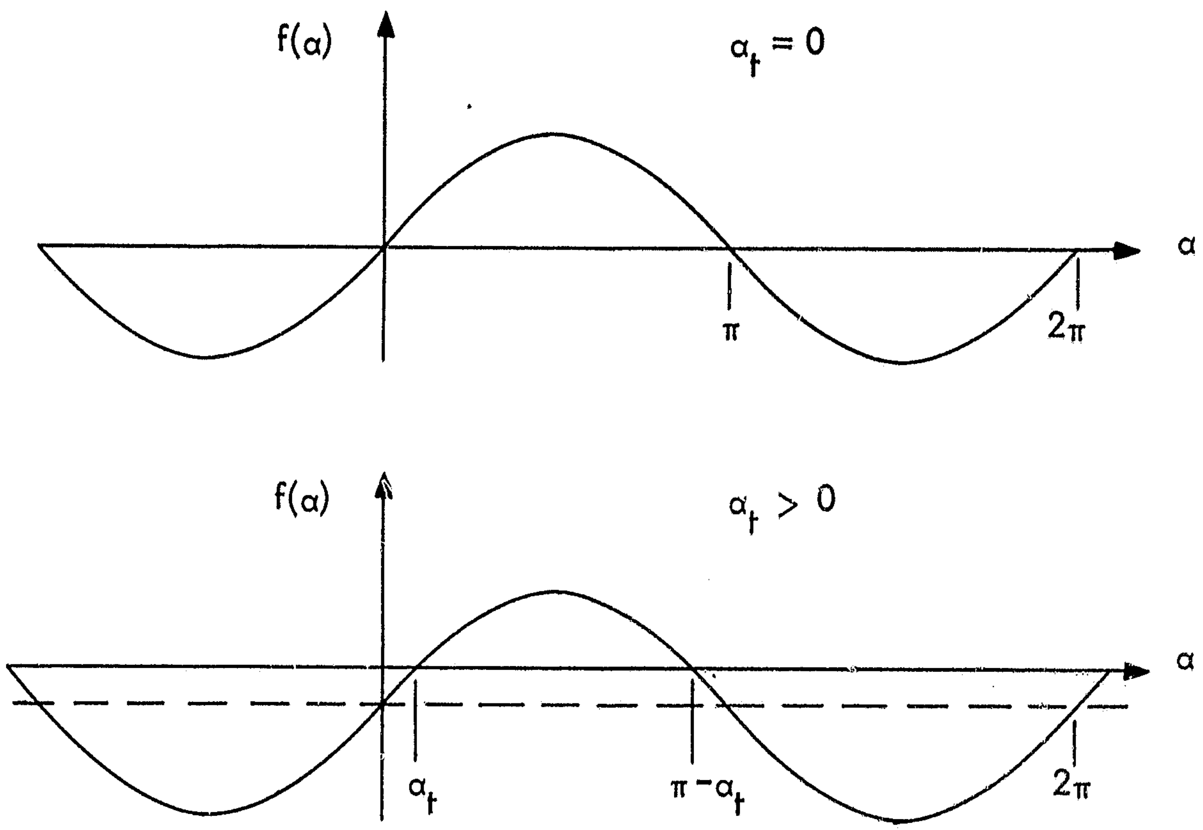
where C_{m_o} is a constant that is an aerodynamic characteristic of a particular body. Tobak assumed a sine function for $f(\alpha)$:

$$f(\alpha) = \sin \alpha \quad . \tag{2}$$

For this symmetric moment with zero trim, C_{m_o} corresponds to $C_{m_{max}}$. This study is concerned with a generalization of expression (2) that has the form

$$f(\alpha) = \sin \alpha - \sin \alpha_t \quad . \tag{3}$$

Here α_t is the stable trim angle, and $\pi - \alpha_t$ is the unstable trim angle. This functional form is shown in sketch 1.



Sketch 1: Expression (3) for $\alpha_t = 0$ and $\alpha_t > 0$.

Following Tobak, we replace the time t by the variable $x = \kappa \exp(st/2)$, where

$$s \equiv bv_i$$

$$\left(\frac{\kappa s}{2}\right)^2 \equiv -q_i \left(\frac{A\ell}{I}\right) C_{m_o} . \qquad (4)$$

Here v_i is the initial vertical component of the flight velocity, q_i the initial dynamic pressure, A and ℓ the reference area and length, and I the pitching moment of inertia about the center of gravity. In these variables the governing equation for the angle of attack α is

$$\frac{d^2\alpha}{dx^2} + \frac{1}{x}\frac{d\alpha}{dx} + f(\alpha) = 0 \qquad (5)$$

The initial conditions for the initial angle α_i and the initial pitching rate $\dot{\alpha}_i$ are

$$\alpha(\kappa) = \alpha_i$$

$$\kappa\left(\frac{d\alpha}{dx}\right)_\kappa = \frac{2\dot{\alpha}_i}{s} \equiv \frac{1}{a} \tag{6}$$

For $f = \sin\alpha$, equation (5) is the equation for Painlevé's fifth transcendent, originally derived by Tobak. Our goal is to study the corresponding asymmetric problem where $f(\alpha)$ is given by (3). Thus the differential equation will contain the trim angle α_t as a parameter, and all the other parameters appear in the initial conditions.

ANALYSIS

The tumbling motion thar occurs before the onset of oscillation can be studied by the analysis used in reference 4. This is done by first recasting equation (5) as an integral equation. Multiplying (5) by $2x^2$ $(d\alpha/dx)$ and integrating once yields

$$\left(\frac{d\alpha}{dx}\right)^2 = \frac{1}{a^2 x^2} \left[1 - 2a^2 \int_{\alpha_i}^{\alpha} x^2 f(\alpha) d\alpha\right] . \qquad (7)$$

The initial conditions (6) were used in this step.

Another integration gives

$$x = \kappa \exp\left[a \int_{\alpha_i}^{\alpha} \frac{d\tilde{\alpha}}{F(\tilde{\alpha};x)}\right] , \qquad (8)$$

where

$$F(\alpha;x) \equiv \left[1 - 2a^2 \int_{\alpha_i}^{\alpha} x^2 f(\tilde{\alpha}) d\tilde{\alpha}\right]^{1/2} . \qquad (9)$$

In integral-equation form, it is straight forward to analyze the problem by means of successive approximations. A first approximation for x that is valid as $\alpha \rightarrow \alpha_i$ can be obtained by neglecting the integral in (9). We obtain

$$x_o(\alpha) = \kappa \exp\left[a(\alpha - \alpha_i)\right] . \qquad (10)$$

It follows that the nth approximation can then be computed by means of the expression

$$x_n = \kappa \exp\left[a \int_{\alpha_i}^{\alpha} \frac{d\tilde{\alpha}}{F(\tilde{\alpha};x_{n-1})}\right] . \qquad (11)$$

If it is assumed that each successive term contributed by the successive - approximation scheme is smaller that the previous term, then it is possible to obtain the formal asymptotic expansion

$$x = x_o \; [1 + x_o^2 \; G_1(\alpha; a, \kappa, \alpha_i) + x_o^4 G_2(\alpha; a, \kappa, \alpha_i) \;\; + \;\; \ldots \;] \qquad , \tag{12}$$

where G_1, G_2, ... are functions pertaining to the respective approximations. The functions G_1, G_2, ... will depend also on any parameters that appear in $f(\alpha)$.

This approximation was shown by Rasmussen (4) to give a good approximation for $f(\alpha) = \sin\alpha$ near tumbling arrest, at least for the minimum angle of pitch that occurs at tumbling arrest. In an analogous manner, we wish to examine the case of nonzero trim by assuming the asymmetric form for $f(\alpha)$ given by (3). This form contains the single parameter α_t.

Special Case for Nonzero Trim

We now assume that $f(\alpha)$ is given by

$$f(\alpha) = \sin\alpha - \sin\alpha_t .$$

Substituting into (9) with $x = x_o(\alpha)$, we obtain as a first approximation

$$F^2(\alpha; x_o) = 1 + a x_o^2 \; [\sin\alpha_t \; - \; \cos\beta \sin(\alpha - \beta)] - a\kappa^2 \;\; [\sin\alpha_t \;\; - \cos\beta \sin(\alpha_i - \beta) \;] \;\; , \tag{13}$$

where $\cot\beta \equiv 2a$. Recall that in the zeroth approximation the function F was set equal to unity. The second and third terms in (13) are thus to be regarded as small corrections. Treating these terms as small, we can expand accordingly and obtain

$$\int_{\alpha_i}^{\alpha} \frac{d\alpha}{F(\alpha;x_o)} = (\alpha - \alpha_i) + \frac{1}{a} x_o^2 G_1(\alpha;a,\kappa,\alpha_i,\alpha_t), \tag{14}$$

where

$$G_1(\alpha;a,\kappa,\alpha_i,\alpha_t) \equiv \frac{a}{4}\left[\cos^2\beta\left\{\sin(\alpha-2\beta) - \frac{\kappa^2}{x_o^2}\sin(\alpha_i-2\beta)\right\} - \left(1-\frac{\kappa^2}{x_o^2}\right)\sin\alpha_t + \frac{\kappa^2}{x_o^2}\ln\frac{x_o^2}{\kappa^2}\left\{\sin\alpha_t - \cos\beta\sin(\alpha_i-\beta)\right\}\right] \tag{15}$$

and the higher-order terms in the expansion have been omitted. Treating the second term in (14) as small, we substitute (14) into (8) and expand again to obtain the first approximation for x as

$$x_1 = x_o[1+x_o^2 G_1(\alpha;a,\kappa,\alpha_i,\alpha_t)]. \tag{16}$$

Repeating this procedure generates the expansion indicated by (12). For $\alpha_t=0$, the above results reduce to those of Rasmussen (4).

It is now convenient to consider the limit $\kappa \to 0$ with x_o held finite. This is consistent with the calculations performed by Tobak and Peterson (3). Because of the exponential law assumed for the atmospheric density, this limit corresponds to the initial conditions being imposed outside the atmosphere (that is, at infinity). For some finite value of x the pitch angle α is regarded as some finite value. Thus the initial value α_i becomes infinite (very large either positively or negatively) and drops out of the problem except as it appears in the function x. Thus in the limit $\kappa \to 0$, we have

$$G_1(\alpha;a,o,\alpha_i,\alpha_t) = \frac{a}{4}[\cos^2\beta\sin(\alpha-2\beta) - \sin\alpha_t] \quad . \tag{17}$$

Because in this limit G_1 does not depend on α_i, we shall denote it by $G_1(\alpha;a,\alpha_t)$.

The second approximation is obtained by squaring (16) and omitting higher-order terms so that

$$x_1^2 = x_o^2 \left[1 + 2x_o^2 G_1\right].$$

The second approximation for the function F defined by (?) is thus

$$F^2\ (\alpha, x_1) = F^2\ (\alpha, x_o) + I_o \quad , \tag{18}$$

where

$$I_o \equiv 4a^2 \int_{\alpha_i}^{\alpha} x_o^2\, G_1\, (\sin\alpha_t - \sin\tilde{\alpha})d\tilde{\alpha} \tag{19}$$

and $F^2(\alpha, x_o)$ is given by (13). Substituting (17) into (19) and taking the limit $\kappa \to 0$ gives

$$I_o = \frac{a^2 x_o^4}{4} \left[(1/2)\cos^3\beta \cos(2\alpha - 3\beta) - (1/2)\cos^2\beta \cos 2\beta + \sin\alpha_t \left\{J + 2\cos^2\beta \sin(\alpha - 2\beta)\right\} - \sin^2\alpha_t\right], \tag{20}$$

where the function J is defined as

$$J \equiv \cos\gamma \left\{\sin(\alpha - \gamma) + \cos^2\beta \sin(\alpha - 2\beta - \gamma)\right\} - 2\cos^2\beta \sin(\alpha - 2\beta), \tag{21}$$

and $\cot\gamma \equiv 4a = 2\cot\beta$. Taking into account the relation between γ and β, we can rewrite J as the simpler form

$$J = \sin\beta \cos\gamma \cos(\alpha - \beta - \gamma). \tag{22}$$

For our purposes in studying tumbling arrest it is not necessary to carry the second approximation to completion and determine x_2. Instead it is useful to rewrite equation (7) in terms of the second approximation for $\kappa \rightarrow 0$. We now have

$$a^2x^2\left(\frac{d\alpha}{dx}\right)^2 \simeq F^2(\alpha, x_1)$$

$$\cong 1 + ax_o^2\left[\sin\alpha_t - \cos\beta\,\sin(\alpha-\beta)\right] + \ldots, \tag{23}$$

A better approximation for this equation is obtained if we replace x_o by x. Using (12) to solve for x_o^2 as a function of x^2 by means of an inverse expansion, we obtain

$$x_o^2 = x^2 - 2G_1x^4 + \ldots \tag{24}$$

Substituting (24) into the right side of (23) and collecting like powers of x^2 gives

$$a^2x^2\left(\frac{d\alpha}{dx}\right)^2 = 1 + A(ax^2) + \frac{1}{4}\,B\,(ax^2)^2 + \ldots, \tag{25}$$

where $A \equiv \sin\alpha_t - \cos\beta\,\sin(\alpha-\beta)$,

$$B \equiv (1/2)\cos^2\beta\left\{1 - \cos\beta\,\cos(2\alpha - 3\beta)\right\} + \sin^2\alpha_t + \sin\alpha_t\left\{J - 2\cos\beta\,\sin(\alpha-\beta)\right\} \tag{26}$$

Equation (25) is the basis for our investigation of tumbling arrest. Because of the preceding step involving (21), equation (25) is valid for much larger values of x than might be expected. Confidence in (25) is gained if one realizes that the above expansion procedure produces the exact form of (25) if $f(\alpha)$ is taken as a constant in equation (5), say $f(\alpha) = -\sin\alpha_t$. In this case the equation is linear and can be integrated exactly. In the limit $\kappa \rightarrow 0$, the first three terms on the right side of (25) give the exact result, with $A = \sin\alpha_t$ and $B = \sin^2\alpha_t$, which are special cases of (26) when terms involving α are eliminated. On the other hand, when $\alpha_t = 0$, Rasmussen (4) has shown that (25) leads to good results for the minimum pitch angle at tumbling arrest. In an analogous fashion, we wish to examine tumbling arrest for $\alpha_t \neq 0$.

Tumbling Arrest

Along with Tobak and Peterson (3), we shall define tumbling arrest as that condition when $(d\alpha/dx)$ first vanishes, that is, at the first peak of oscillatory motion. Let x_p and α_p denote conditions at $(d\alpha/dx) = 0$. The relation between x_p and α_p can be obtained by setting the right-hand side of (25) equal to zero and solving for x_p^2. Recalling that $2 \cot\beta = \cot\gamma = 4a = \frac{2s}{\dot{\alpha}_i}$, we obtain

$$x_p^2 = \frac{4 \tan\beta \sec\beta}{\sin(\alpha_p - \beta) - \sec\beta \sin\alpha_t \pm \left[(1/2)\left\{\sin\beta \sin(2\alpha_p - 3\beta) - 2\sin\alpha_t \sec^2\beta \, J_p\right\}\right]^{1/2}} \tag{27}$$

where

$$J_p = \sin\beta \cos\gamma \cos(\alpha_p - \beta - \gamma).$$

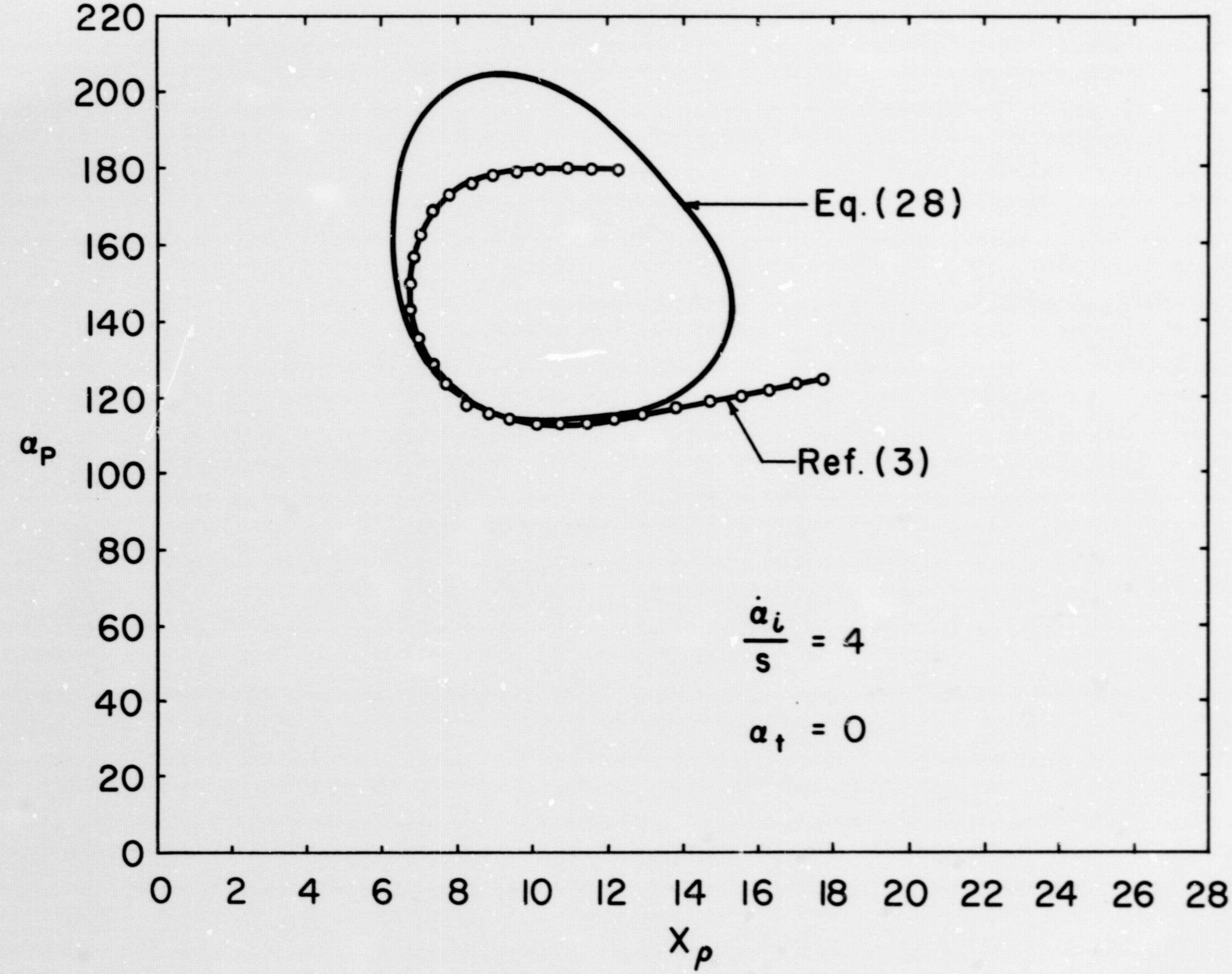
For $\alpha_t = 0$, this result reduces to that of Rasmussen (4).

Zero Trim

The case of zero trim angle is of particular interest because our present results can be compared with the numerical calculations of Tobak and Peterson (3). For $\alpha_t = 0$, expression (27) reads

$$x_p^2 = \frac{4 \tan\beta \sec\beta}{\sin(\alpha_p - \beta) \pm \left[(1/2)\sin\beta \sin(2\alpha_p - 3\beta)\right]^{1/2}} \tag{28}$$

This expression is plotted in Fig. 1 for $\frac{\dot{\alpha}_i}{s} = 4$. For a given value of α_p,



Angle of Arrest as a Function of X_ρ. Zero Trim.

Fig. 1.

x_p has two values corresponding to the two branches of (28) that are produced by the plus and minus signs in the denominator. Also shown in Fig. 1 are numerical points taken from the curves of Tobak and Peterson (3). In the region of the minimum value of α_p the agreement of (28) and the calculations of Tobak and Peterson is good. The prediction of the minimum value of x_p is fair. Elsewhere the prediction of expression (28) breaks down.

The comparison for other values of $\dot{\alpha}/s$ can be easily illustrated for the minimum values of α_p, denoted by $\alpha_{p_{min}}$. The minimum value of α_p occurs when the square-root term in the denominator of (28) vanishes. In this case we obtain

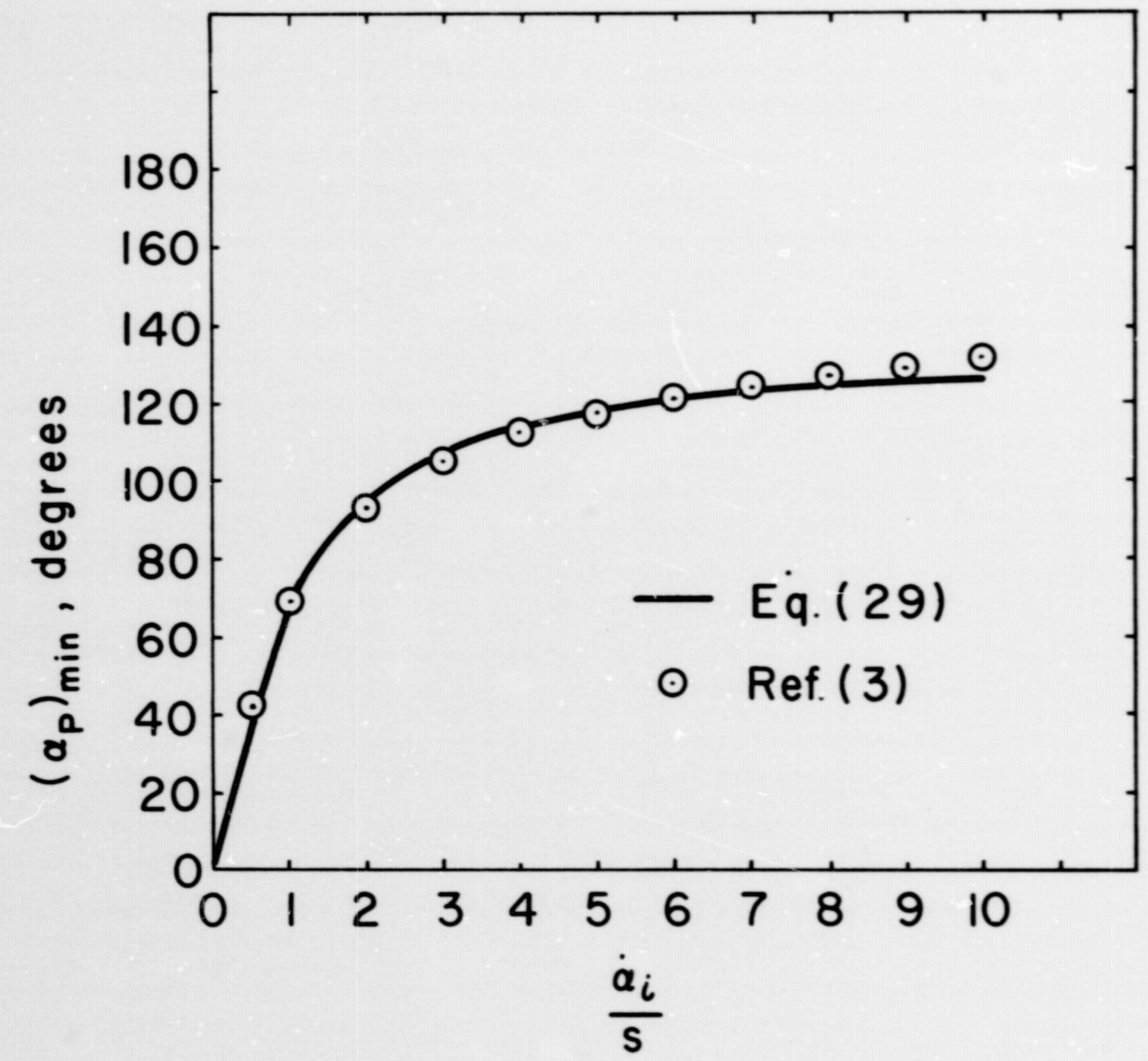
$$\alpha_{p_{min}} = \frac{3}{2}\beta = \frac{3}{2}\tan^{-1}\frac{\dot{\alpha}_i}{s} \quad . \tag{29}$$

Expression (29) is compared with the numerical data of Tobak and Peterson in Fig. 2. Agreement is quite good except for large values of $\dot{\alpha}_i/s$. The comparison for the value of x_p that corresponds to $\alpha_{p_{min}}$ is shown in Fig. 3. Again the agreement with the data of Tobak and Peterson is good except for large values of $\dot{\alpha}_i/s$.

Another situation of interest occurs when x_p is a minimum. The result for the present analysis is obtained from (28) by taking the derivative of x_p with respect to α_p and setting it equal to zero. After some manipulation we obtain the result

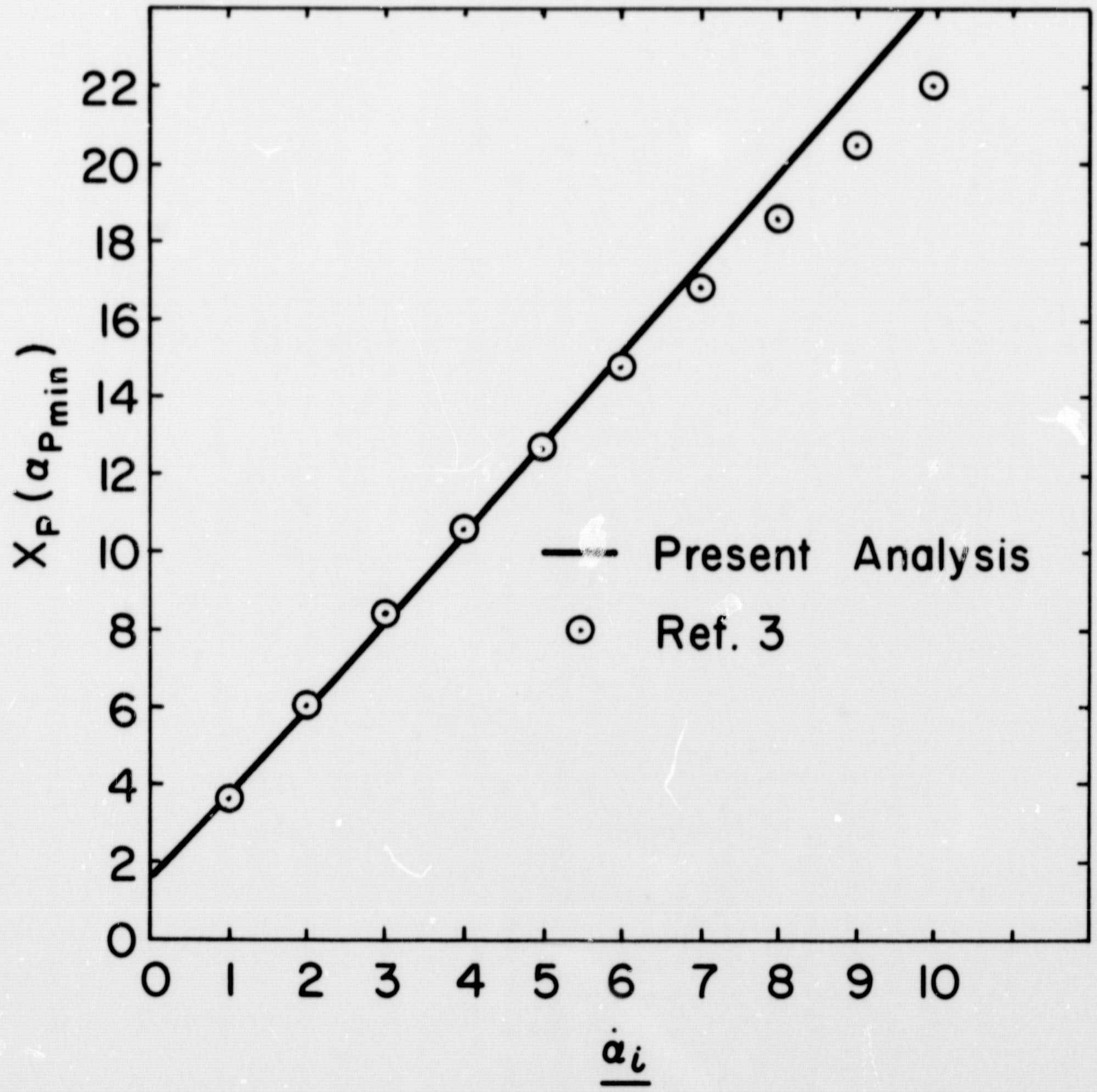
$$\sin(2\alpha_p - 3\beta)[1 + \cos\beta\cos(2\alpha_p - 3\beta)] = \sin\beta \quad . \tag{30}$$

In Fig. 4, the value of α_p is plotted as a function of $\dot{\alpha}_i/s = \tan\beta$ as determined by expression (30). The data taken from Tobak and Peterson are also



$(\alpha_p)_{min}$ as a function of initial angular velocity, $\alpha_t = 0$

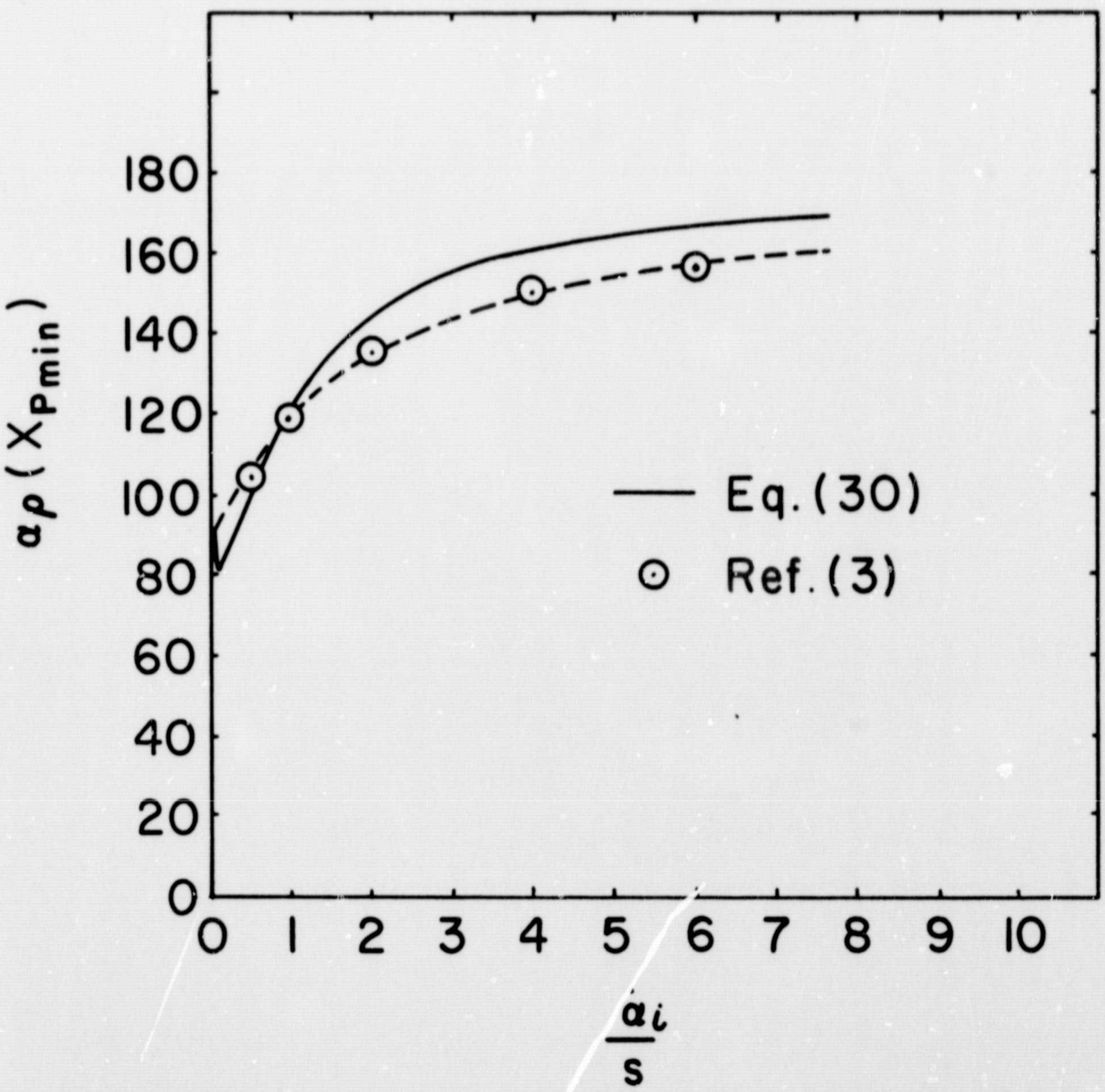
Fig. 2.



$X_\rho(\alpha_{P_{min}})$ as a function of initial angular velocity,

$\alpha_t = 0$.

Fig. 3.



$\alpha_\rho(X_{P_{min}})$ as a function of initial anglar velocity,

$\alpha_t = 0$

Fig. 4.

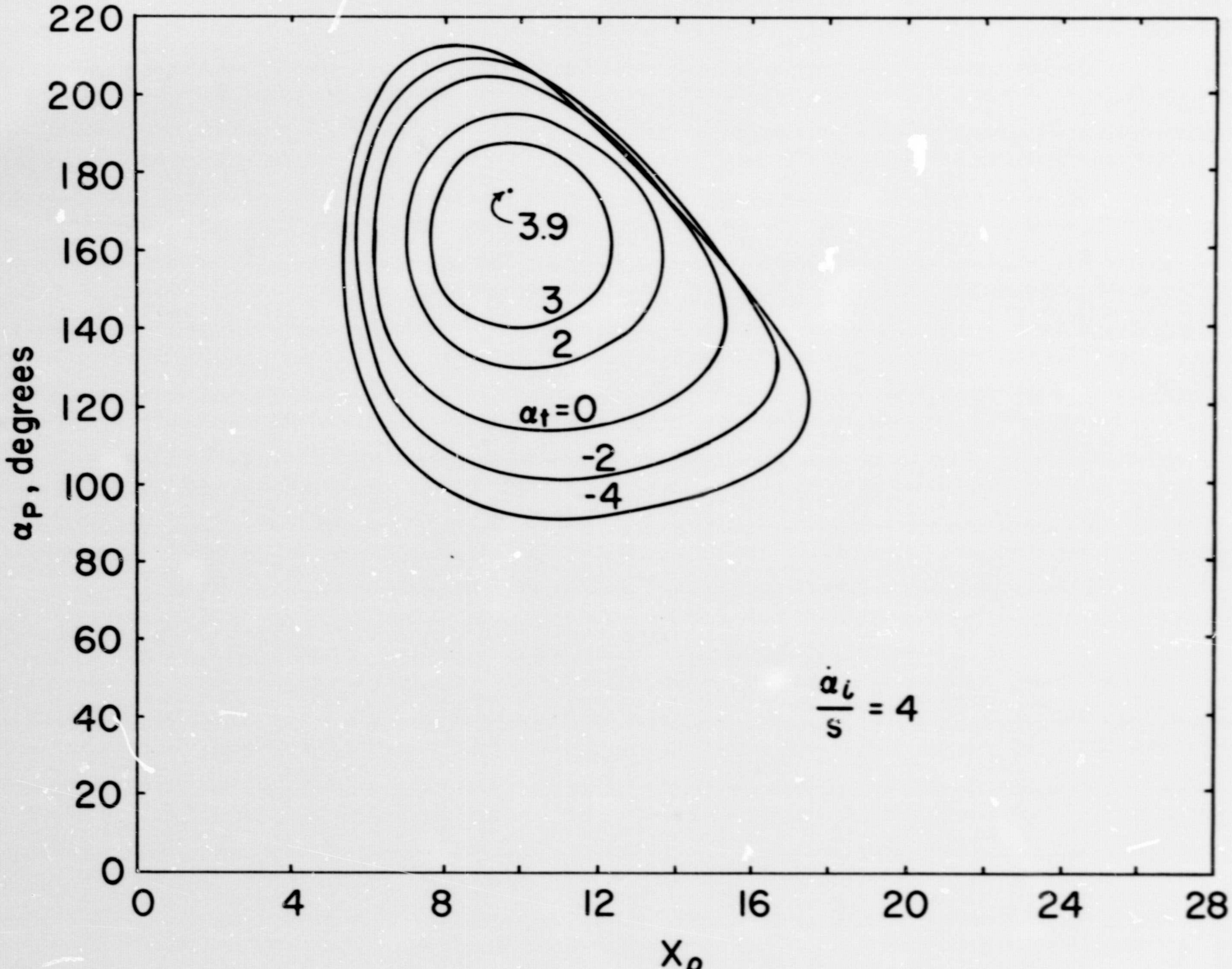
shown. The agreement is not as good as for $\alpha_{p_{min}}$, but it shows that the present approximation is qualitatively correct for the minimum value of x_p, while showing fair quantitative agreement.

The above results for zero trim show that the present analysis is good in the region of minimum α_p and x_p. It is reasonable to expect that this will also be true for non-zero trim, at least when α_t is sufficiently close to zero. In connection with this it is worth mentioning that if $f(\alpha)$ is a constant in (5), say $f(\alpha) = -\sin \alpha_t$, then the expansion procedure of this analysis generates the exact result for equation (25), with $A = \sin \alpha_t$ and $B = \sin^2 \alpha_t$. This fact together with the agreement with numerical calculations for zero trim provide confidence in the results for nonzero trim.

Nonzero Trim

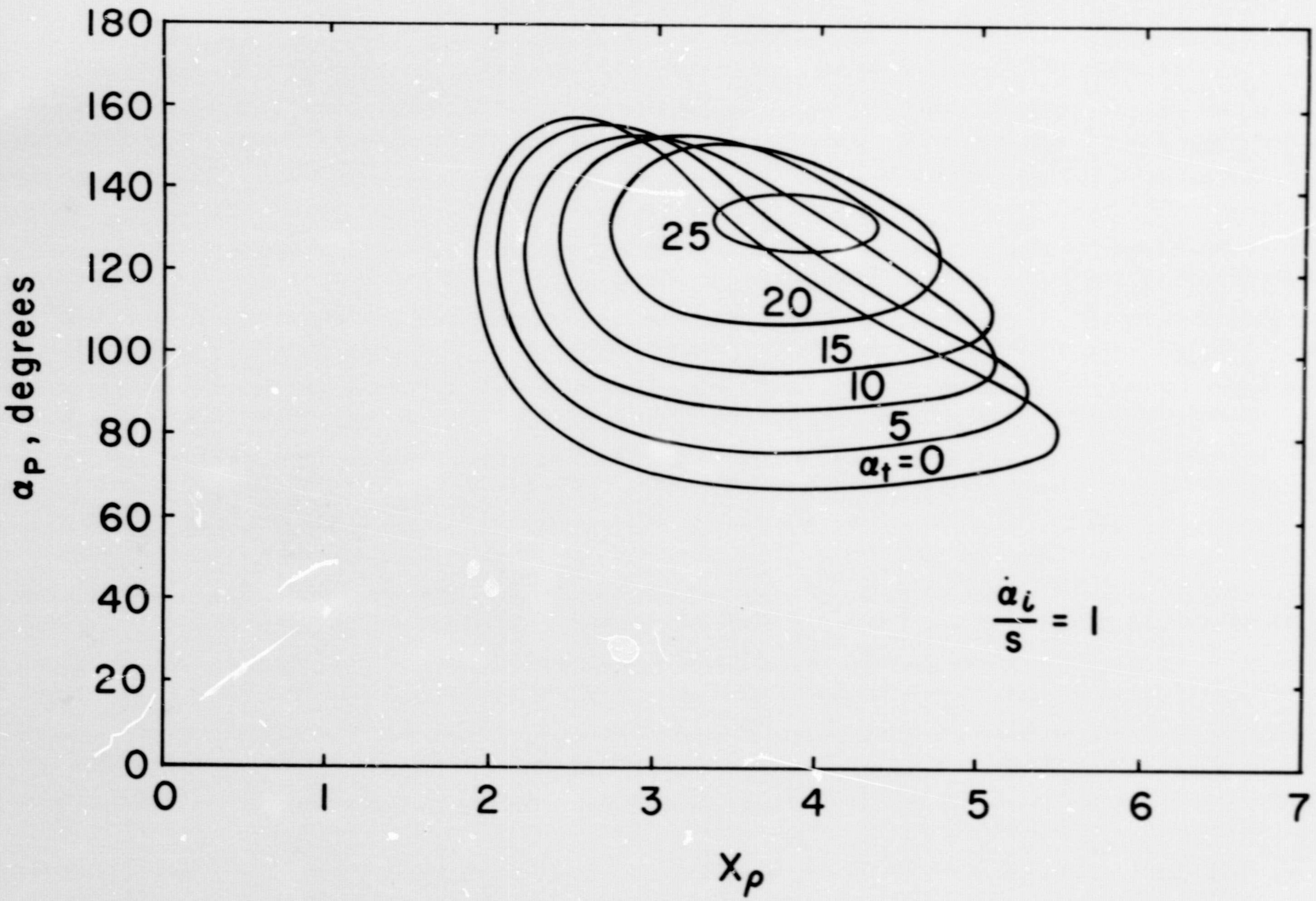
The angle of arrest α_p is plotted as a function of x_p in Fig. 5 for various values of α_t, as determined by expression (27). We selected $\dot{\alpha}_i/s = 4$ in accordance with the example for zero trim shown in Fig. 1. For nonzero trim, a limiting case occurs when α_t is greater than zero. As can be seen from Fig. 5, the closed curves become smaller as α_t increases, and finally shrink to a point when $\alpha_t = 3.9^{\circ}$. This value represents a limiting condition for $\dot{\alpha}_i/s = 4$. When α_t exceeds 3.9°, tumbling will never cease, and the tumbling rate will begin to increase after reaching a minimum value. When α_t is negative, the closed curves become larger as α_t becomes more negative. The curves begin to cross each other in the upper right-hand portion of Fig. 5. No physical significance can be attached to this since the analysis is not valid in this region. When the sign of $\dot{\alpha}_i/s$ is reversed, the same figure applies for reversed signs on α_p and α_t.

Fig. 6 shows a similar plot for $\dot{\alpha}_i/s = 1$. Again the closed curves become smaller as α_t increases. In this case the limiting value of α_t is $\alpha_t = 26^{\circ}$. When



Angle of arrest as a function of X_ρ. Nonzero trim, $\frac{\dot{\alpha}_i}{s} = 4$.

Fig. 5.



Angle of arrest as a function of X_ρ. Nonzero trim, $\frac{\dot{\alpha}_i}{s} = 1$.

Fig. 6.

α_t is greater than 26°, tumbling will never cease, and the tumbling rate will increase after it reaches a minimum. The curves for α_t less than zero are not shown for clarity. There is no limiting condition for $\alpha_t < 0$, and tumbling arrest always occurs.

It is of some interest to determine the relationship between α_t and $\beta = \tan^{-1}\left(\dot{\alpha}_i/s\right)$ that separates the regime in which tumbling will always be arrested from that in which tumbling will not be arrested. As can be deduced from perusing Figs. 5 and 6, the limiting case for tumbling arrest occurs when the square-root term in Eq. (27) vanishes, that is,

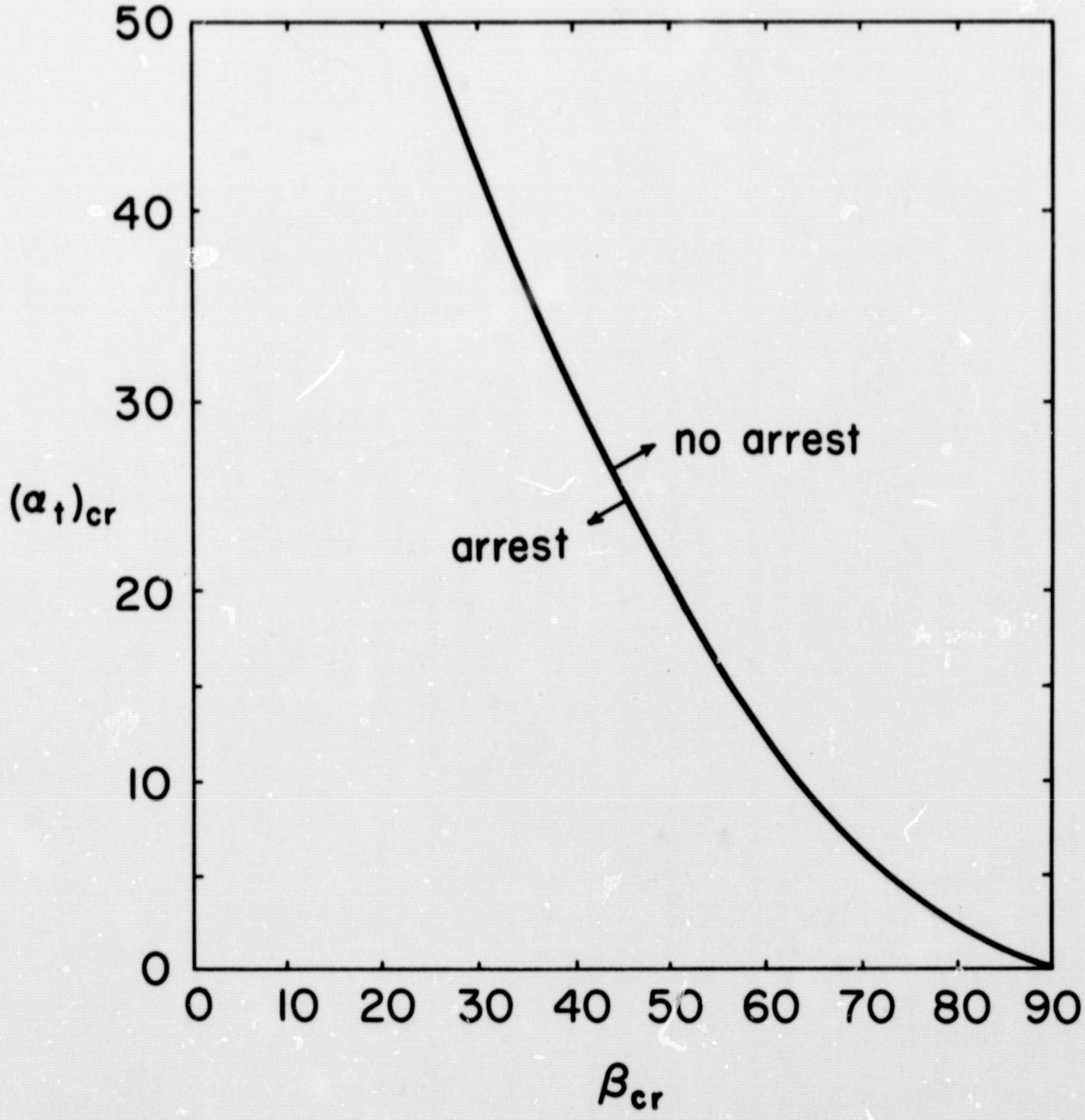
$$\sin(2\alpha_p - 3\beta) - 2\sin\alpha_t \sec^2\beta \cos\gamma \cos(\alpha_p - \beta - \gamma) = 0, \tag{31}$$

and when α_t is a maximum when β is held constant. Thus from Eq. (31), we wish to determine α_t as a function of β when

$$\left(\frac{\partial\alpha_t}{\partial\alpha_p}\right)_\beta = 0.$$

Because of the transcendental nature of Eq. (31), an explicit analytical formula cannot be obtained. We thus proceed in the following manner. By means of (31), we plot α_t as a function of α_p for various values of β held constant. The maximum value of α_t is easy to obtain, for each value of β, from these curves. The values of $(\alpha_t)_{cr}$ and β_{cr} obtained in this manner are shown in Fig. 7. The value of $(\alpha_t)_{cr}$ decreases as $(\beta)_{cr}$ decreases. The region below the curve corresponds to situations in which tumbling arrest always occurs. When α_t and β correspond to a point that falls above the curve, tumbling arrest will not occur, and the tumbling rate will eventually increase as the vehicle penetrates deeper into the atmosphere. When β is negative the corresponding results hold when α_t is also negative.

When β is positive and α_t is negative, or vice versa, tumbling arrest always occurs. When α_t is zero, oscillatory motion always occurs subsequent to tumbling



$(\alpha_t)_{cr}$ as a function of β_{cr}

Fig. 7.

arrest. When α_t is negative (and β positive) the possibility exists that the vehicle may oscillate after arrest, or, if the trim moment is strong enough, to begin tumbling in the opposite direction. The analysis developed in this investigation is not capable of delineating these two possibilities.

CONCLUDING REMARKS

An analysis has been presented that investigates the effect of nonzero trim on the planar tumbling motion of a vehicle undergoing atmospheric entry. A simple extension of the zero-trim model of Tobak (1) and Tobak and Peterson (3), was used to investigate the motion. When the initial angular tumbling rate is positive ($\beta > 0$) and the trim angle positive ($\alpha_t > 0$), or vice versa, there is a critical value of α_t for a given value of β for which tumbling will not be arrested when $\alpha_t > (\alpha_t)_{cr}$. For the situation when α_t is negative and β positive (or vice versa), the present analysis is not capable of determining when tumbling in the reverse direction follows tumbling arrest instead of oscillatory motion which is typical of the zero-trim angle case. Further investigation should be directed toward this end.

The numerical values obtained in this investigation suffer because of the approximate nature of the analysis. Nevertheless, their accuracy was demonstrated for certain limiting cases, and the results should represent, at least qualitatively, the nature of the tumbling motion. Moreover, these results should serve as a basis for a more precise numerical analysis.

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